AP CALCULUS AB
DR. PAUL L. BAILEY

Homework 4.4a
Tuesday, January 7, 2020

Name: KEY

Write your homework *neatly, in pencil,* on blank white $8\frac{1}{2} \times 11$ printer paper. Always *write the problem,* or at least enough of it so that your work is readable. When appropriate, *write in sentences.*

The main theory of section 4.4 is summarized below. Note that our definition for "concave up" and "concave down" is more general than that in Thomas.

Definition 1. Let f be a function defined on an interval I. Let $x_1, x_2 \in I$ with $x_1 < x_2$. The *chord* from $(x_1, f(x_1))$ to $(x_2, f(x_2))$ is the function

$$k : [x_1, x_2] \to \mathbb{R} \quad \text{given by} \quad k(x) = \frac{f(x_2) - f(x_1)}{x_2 - x_1}(x - x_1) + f(x_1).$$

We say that f is *concave up on* I if $k(x) \geq f(x)$ for all $x_1, x_2 \in I$.
We say that f is *concave down on* I if $k(x) \leq f(x)$ for all $x_1, x_2 \in I$.

Thus, the graph of k is the line segment from $(x_1, f(x_1))$ to $(x_2, f(x_2))$; f is concave up if this line segment always lies above the graph of f, and f is concave down if this line segment always lies below the graph of f.

Definition 2. Let f be defined in an interval containing c.
We say that f has a *point of inflection* at c if the graph of f has a tangent line at c, and the concavity of f changes at c.

Theorem 1. (Second Derivative Test for Concavity)
Let f be twice differentiable at c and suppose $f'(c) = 0$.

- *If $f''(c) > 0$, then f has a local minimum at c.*
- *If $f''(c) < 0$, then f has a local maximum at c.*

Theorem 2. (Second Derivative Test for Local Extrema)
Let f be twice differentiable on an open interval I.

- *If $f'' > 0$ on I, then f is concave up on I.*
- *If $f'' < 0$ on I, then f is concave down on I.*

Problem 1. Let $f(x) = x^2 - 4x + 3$. Find intervals on which f is concave up and concave down.

Problem 2. Let $f(x) = x^3 - 3x + 3$. Find intervals on which f is concave up and concave down.

Problem 3. Let $f(x) = x^4 - 2x^2$. Find intervals on which f is concave up and concave down.

Problem 4. Let

$$f(x) = x^4 - 10x^2 + 9.$$

(a) Find intervals on which f is increasing and decreasing.

(b) Find intervals on which f is concave up and concave down.

Problem 5. Let

$$f(x) = \begin{cases} x - n & \text{if } x \in [n, n+1) \text{ for an even integer } n \\ n - x + 1 & \text{if } x \in [n, n+1) \text{ for an odd integer } n \end{cases}$$

Find intervals on which f is concave up and concave down.

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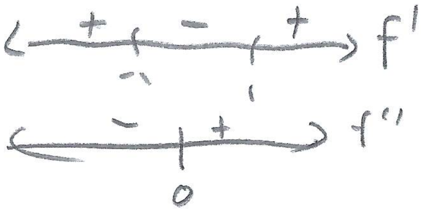
#1 let $f(x) = x^2 - 4x + 3$

$f'(x) = 2x - 4$ So f is concave up on all of $\mathbb{R}$

$f''(x) = 2 > 0$

#2 $f(x) = x^3 - 3x + 3$

$f'(x) = 3x^2 - 3 = 3(x^2 - 1) = 3(x-1)(x+1)$



$f''(x) = 6x$

Concave up on $(0, \infty)$

Concave down on $(-\infty, 0)$

#3 let $f(x) = x^4 - 2x^2 = x^2(x^2 - 2)$

$f'(x) = 4x^3 - 4x$

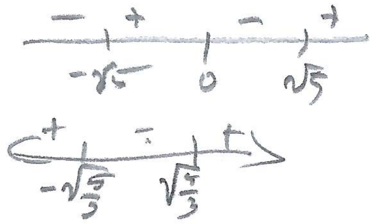
$f''(x) = 12x^2 - 4 = 0 \Rightarrow x = \sqrt{\frac{1}{3}}$

concave down on $(-\infty, \sqrt{\frac{1}{3}})$

Concave up on $(\sqrt{\frac{1}{3}}, \infty)$

#4 let $f(x) = x^4 - 10x^2 + 9$

a) $f'(x) = 4x^3 - 20x = 4x(x^2 - 5)$



$f''(x) = 12x^2 - 20 = 4(3x^2 - 5)$

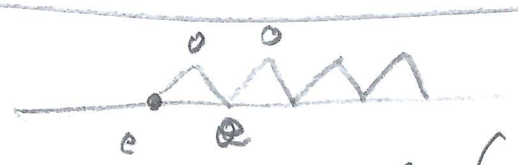
incr on $(-\sqrt{5}, 0) \cup (\sqrt{5}, \infty)$

decr on $(-\infty, -\sqrt{5}) \cup (0, \sqrt{5})$

conc. down on $(-\sqrt{\frac{5}{3}}, \sqrt{\frac{5}{3}})$

conc up on $(-\infty, -\sqrt{\frac{5}{3}}) \cup (\sqrt{\frac{5}{3}}, \infty)$

#5 ~~Conc~~



Concave up on interval $(n, n+2)$ when n is even

Concave down on $(n, n+2)$ where n is odd.

#6) Let $f : [0, 4\pi] \to \mathbb{R}$ by $f(x) = \sin x$

Critical points at
$x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}$

Poi $x = \pi, 2\pi, 3\pi$

incre on $(0, \frac{\pi}{2}) \cup (\frac{3\pi}{2}, \frac{5\pi}{2}) \cup (\frac{7\pi}{2}, 4\pi)$

decre on $(\frac{\pi}{2}, \frac{3\pi}{2}) \cup (\frac{5\pi}{2}, \frac{7\pi}{2})$

conc up on
$(\pi, 2\pi) \cup (3\pi, 4\pi)$
down:
$(0, \pi) \cup (2\pi, 3\pi)$

#7) $\log_b x = \frac{\log_a x}{\log_a b}$. So, $\frac{d}{dx} \log_{10} x = \frac{d}{dx} \frac{\ln x}{\ln b} = \frac{1}{(\ln b) x}$

#8) $f(x) = \frac{e^x + e^{-x}}{2}$ $\quad g(x) = \frac{e^x - e^{-x}}{2}$

(a) So $f'(x) = \frac{1}{2}[e^x - e^{-x}] = g(x)$ and $g'(x) = \frac{1}{2}[e^x - (-e^{-x})] = f(x)$

(b) show $f(2x) = f(x)^2 + g(x)^2$

$F(2x) = \frac{1}{2}(e^{2x} + e^{-2x}]$

$f(x)^2 + g(x)^2 = \frac{1}{2}[e^{2x} + 2 + e^{-2x}] + \frac{1}{2}[e^{2x} - 2 + e^{-2x}]$

$= \frac{1}{2}[e^{2x} + e^{-2x}] = f(2x)$

#9) Let $f(x) = x^3 + ax^2 + bx = x(x^2 + ax + b)$ $\quad a, b \neq 0$

$f'(x) = 3x^2 + 2ax + b$

$f(x) = 0 \Rightarrow x = 0$ or $x = \frac{-a \pm \sqrt{a^2 - 4b}}{2}$

So f has exactly 3 zeros if $a^2 > 4b$
2 if $a = 4b$

$f'(x) = 0 \Rightarrow \Delta = 4 - 12b$. f' has exactly 2 zeros if $4 > 12b$
1 if $4 = 12b$
0 if $4 < 12b$